**Appendix A: Decision Models and Mathematical Forms**

Table A1. Summary of risky decision models.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ID | Model | Category | Authors | Year | Source (Journal or Book) |
| 1 | Expected value maximization | SEUT |  |  |  |
| 2 | Expected utility maximization | SEUT | Bernoulli | 1738 | Papers of the Imperial Academy of Sciences in Petersburg |
| 3 | Subjective expected utility | SEUT | Savage | 1954 | The Foundations of Statistics |
| 4 | Subjective expected money | SEUT | Edwards | 1955 | Journal of Experimental Psychology |
| 5 | Certainty equivalence theory | SEUT | Handa | 1977 | Journal of Political Economy |
| 6 | Odds-based subjective weighted utility | SEUT | Karmarkar | 1978 | Organizational Behavior and Human Performance |
| 7 | Prospect theory | SEUT | Kahneman and Tversky | 1979 | Econometrica |
| 8 | Dual theory w/ hyperbolic weighting | SEUT | Yaari | 1987 | Econometrica |
| 9 | Dual theory w/ quadratic weighting | SEUT | Yaari | 1987 | Econometrica |
| 10 | Prospective reference theory | SEUT | Viscusi | 1989 | Journal of Risk and Uncertainty |
| 11 | Venture theory | SEUT | Hogarth and Einhorn | 1990 | Management Science |
| 12 | Cumulative prospect theory | SEUT | Tversky and Kahneman | 1992 | Journal of Risk and Uncertainty |
| 13 | Cumulative prospect theory w/ Lattimore et al.’s weighting | SEUT | Lattimore et al. | 1992 | Journal of Economic Behavior and Organization |
| 14 | Decision field theory | SEUT | Busemeyer and Townsend | 1993 | Psychological Review |
| 15 | Rank affected multiplicative weighting | SEUT | Birnbaum | 1997 | Choice, Decision, and Measurement: Essays in Honor of R. Duncan Luce |
| 16 | Cumulative prospect theory w/ Prelec’s weighting | SEUT | Prelec | 1998 | Econometrica |
| 17 | Cumulative prospect theory w/ Gonzalez and Wu’s weighting | SEUT | Gonzalez and Wu | 1999 | Cognitive Psychology |
| 18 | Prospect theory w/ Wu et al.’s editing rule | SEUT | Wu et al. | 2005 | Journal of Risk and Uncertainty |
| 19 | Transfer of attention exchange | SEUT | Birnbaum | 2008 | Psychological Review |
| 20 | Dual systems w/ expected value evaluation | SEUT | Mukherjee | 2010 | Psychological Review |
| 21 | Salience theory | SEUT | Bordalo et al. | 2012 | Quarterly Journal of Economics |
| 22 | Distracted decision field theory | SEUT | Bhatia | 2014 | Psychonomic Bulletin & Review |
| 23 | Dual systems w/ expected utility evaluation | SEUT | Loewenstein et al. | 2015 | Decision |
| 24 | Noisy retrieval | SEUT | Marchiori et al. | 2015 | Decision |
| 25 | Utility-weighted sampling | SEUT | Lieder et al. | 2018 | Psychological Review |
| 26 | Portfolio theory w/ variance | Risk-as-value | Markowitz | 1952 | Journal of Finance |
| 27 | Mean, variance and skewness | Risk-as-value | Coombs and Pruitt | 1960 | Journal of Experimental Psychology |
| 28 | Alpha target model | Risk-as-value | Fishburn | 1977 | American Economic Review |
| 29 | Below target model | Risk-as-value | Fishburn | 1977 | American Economic Review |
| 30 | Portfolio theory w/ standard deviation | Risk-as-value | Fishburn | 1977 | American Economic Review |
| 31 | Below-mean semivariance | Risk-as-value | Fishburn | 1977 | American Economic Review |
| 32 | Below-target semivariance | Risk-as-value | Fishburn | 1977 | American Economic Review |
| 33 | Relative risk-value model w/ general power | Risk-as-value | Dyer and Jia | 1997 | European Journal of Operational Research |
| 34 | Relative risk-value model w/ linear plus power | Risk-as-value | Dyer and Jia | 1997 | European Journal of Operational Research |
| 35 | Relative risk-value model w/ logarithmic | Risk-as-value | Dyer and Jia | 1997 | European Journal of Operational Research |
| 36 | Relative risk-value model w/ multiplicative power | Risk-as-value | Dyer and Jia | 1997 | European Journal of Operational Research |
| 37 | Coefficient of variation | Risk-as-value | Weber | 2004 | Psychological Review |
| 38 | Aspiration-level theory | Risk-as-value | Diecidue and van de Ven | 2008 | International Economic Review |
| 39 | Regret theory w/ expected value evaluation | Counterfactual | Bell | 1982 | Operations Research |
| 40 | Regret theory w/ expected utility evaluation | Counterfactual | Loomes and Sugden | 1982 | Economic Journal |
| 41 | Disappointment theory w/o rescaling | Counterfactual | Bell | 1985 | Operations Research |
| 42 | Disappointment theory w/ expected value evaluation | Counterfactual | Loomes and Sugden | 1986 | Review of Economic Studies |
| 43 | Disappointment theory w/ expected utility evaluation | Counterfactual | Loomes and Sugden | 1986 | Review of Economic Studies |
| 44 | Subjective expected pleasure | Counterfactual | Mellers et al. | 1999 | Journal of Experimental Psychology: General |
| 45 | Generalized disappointment theory w/ expected value evaluation | Counterfactual | Delquié and Cillo | 2006 | Journal of Risk and Uncertainty |
| 46 | Generalized disappointment theory w/ expected utility evaluation | Counterfactual | Delquié and Cillo | 2006 | Journal of Risk and Uncertainty |
| 47 | Better than average | Heuristics | Thorgate | 1980 | Behavioral Science |
| 48 | Consequence count | Heuristics | Thorgate | 1980 | Behavioral Science |
| 49 | Equiprobable | Heuristics | Thorgate | 1980 | Behavioral Science |
| 50 | Low expected payoff elimination | Heuristics | Thorgate | 1980 | Behavioral Science |
| 51 | Least likely | Heuristics | Thorgate | 1980 | Behavioral Science |
| 52 | Low-payoff elimination | Heuristics | Thorgate | 1980 | Behavioral Science |
| 53 | Maximax | Heuristics | Thorgate | 1980 | Behavioral Science |
| 54 | Minimax | Heuristics | Thorgate | 1980 | Behavioral Science |
| 55 | Minimax Regret | Heuristics | Thorgate | 1980 | Behavioral Science |
| 56 | Most likely | Heuristics | Thorgate | 1980 | Behavioral Science |
| 57 | Most probable winner | Heuristics | Thorgate | 1980 | Behavioral Science |
| 58 | Relative expected loss minimization | Heuristics | Thorgate | 1980 | Behavioral Science |
| 59 | Similarity | Heuristics | Rubinstein | 1988 | Journal of Economic Theory |
| 60 | Similarity w/ expected utility evaluation | Heuristics | Leland | 1994 | Journal of Risk and Uncertainty |
| 61 | Priority heuristic | Heuristics | Brandstatter et al. | 2006 | Psychological Review |
| 62 | Perceived relative argument model | Heuristics | Loomes | 2010 | Psychological Review |

Table A2. Functional forms of risky decision models. The notations are designed for choices between and , where ,. , and . denotes the utility or choice propensity of *X* and denotes the utility or choice propensity of *Y*. If not given, can be obtained by replacing and with and respectively in . When involves interactions with or , the corresponding replaces and in with and respectively. For heuristic models, denotes the argument for option *X* and denotes the argument for option *Y*. If not given, , can be obtained by replacing and with and respectively. When involves interactions with or , the corresponding should replace and with and respectively. Free parameters are denoted by Greek letters, with corresponding domains and prior distributions shown in Supplementary Table 5.

|  |  |  |  |
| --- | --- | --- | --- |
| ID | Model | Function | Stochastic specification[[1]](#footnote-1) |
| 1 | Expected value maximization |  | Logit or Probit |
| 2 | Expected utility maximization |  | Logit or Probit |
| 3 | Subjective expected utility |  | Logit or Probit |
| 4 | Subjective expected money |  | Logit or Probit |
| 5 | Certainty equivalence theory |  | Logit or Probit |
| 6 | Odds-based subjective weighted utility |  | Logit or Probit |
| 7 | Prospect theory |  | Logit or Probit |
| 8 | Dual theory w/ hyperbolic weighting |  | Logit or Probit |
| 9 | Dual theory w/ quadratic weighting |  | Logit or Probit |
| 10 | Prospective reference theory |  | Logit or Probit |
| 11 | Venture theory |  | Logit or Probit |
| 12 | Cumulative prospect theory |  | Logit or Probit |
| 13 | Cumulative prospect theory w/ Lattimore et al.’s weighting |  | Logit or Probit |
| 14 | Decision field theory | , where | NA |
| 15 | Rank affected multiplicative weighting |  | Logit or Probit |
| 16 | Cumulative prospect theory w/ Prelec’s weighting |  | Logit or Probit |
| 17 | Cumulative prospect theory w/ Gonzalez and Wu’s weighting |  | Logit or Probit |
| 18 | Prospect theory w/ Wu et al.’s editing rule |  | Logit or Probit |
| 19 | Transfer of attention exchange[[2]](#footnote-2) |  | Logit or Probit |
| 20 | Dual systems w/ expected value evaluation |  | Logit or Probit |
| 21 | Salience theory | , where corresponds to the rank order of the four pairwise salience values . | Logit or Probit |
| 22 | Distracted decision field theory | , where | NA |
| 23 | Dual systems w/ expected utility evaluation |  | Logit or Probit |
| 24 | Noisy retrieval |  | Logit or Probit |
| 25 | Utility-weighted sampling[[3]](#footnote-3) | , where  is the probability of sampling *X* from {*X*, *Y*} and is the binomial probability mass function of sampling *X* for *k* times out of the total times, the latter of which is a free parameter for the model. | NA |
| 26 | Portfolio theory w/ variance |  | Logit or Probit |
| 27 | Mean, variance and skewness |  | Logit or Probit |
| 28 | Alpha target model[[4]](#footnote-4) |  | Logit or Probit |
| 29 | Below target model |  | Logit or Probit |
| 30 | Portfolio theory w/ standard deviation |  | Logit or Probit |
| 31 | Below-mean semivariance |  | Logit or Probit |
| 32 | Below-target semivariance |  | Logit or Probit |
| 33 | Relative risk-value model w/ general power |  | Logit or Probit |
| 34 | Relative risk-value model w/ linear plus power |  | Logit or Probit |
| 35 | Relative risk-value model w/ logarithmic |  | Logit or Probit |
| 36 | Relative risk-value model w/ multiplicative power |  | Logit or Probit |
| 37 | Coefficient of variation |  | Logit or Probit |
| 38 | Aspiration-level theory | , where  is the aspiration level. | Logit or Probit |
| 39 | Regret theory w/ expected value evaluation |  | Logit or Probit |
| 40 | Regret theory w/ expected utility evaluation |  | Logit or Probit |
| 41 | Disappointment theory w/o rescaling |  | Logit or Probit |
| 42 | Disappointment theory w/ expected value evaluation |  | Logit or Probit |
| 43 | Disappointment theory w/ expected utility evaluation |  | Logit or Probit |
| 44 | Subjective expected pleasure |  | Logit or Probit |
| 45 | Generalized disappointment theory w/ expected value evaluation |  | Logit or Probit |
| 46 | Generalized disappointment theory w/ expected utility evaluation |  | Logit or Probit |
| 47 | Better than average |  | Constant-error |
| 48 | Consequence count |  | Constant-error |
| 49 | Equiprobable |  | Constant-error |
| 50 | Low expected payoff elimination |  | Constant-error |
| 51 | Least likely |  | Constant-error |
| 52 | Low-payoff elimination |  | Constant-error |
| 53 | Maximax |  | Constant-error |
| 54 | Minimax |  | Constant-error |
| 55 | Minimax Regret |  | Constant-error |
| 56 | Most likely |  | Constant-error |
| 57 | Most probable winner |  | Constant-error |
| 58 | Relative expected loss minimization | , where | NA |
| 59 | Similarity |  | Constant-error |
| 60 | Similarity w/ expected utility evaluation |  | Constant-error |
| 61 | Priority heuristic |  | Constant-error |
| 62 | Perceived relative argument model |  | Logit or Probit |

*Note*. In order to ensure that the KL divergence between two series of model predictions is tractable, choice probabilities for all models are bounded within the interval [0.001, 0.999]. The following additional functions are used in Supplementary Table 2:

* is a sign function that returns 1 if the argument is positive, -1 if negative and 0 if zero.
* is an indicator function that returns 1 if the argument is true, 0 otherwise.
* Power value function:
* Relative value function for the utility-weighted sampling model:
* Tversky and Kahneman’s (1992) probability weighting function:
* Karmarkar’s (1978) probability weighting function:
* Lattimore et al.’s (1992) probability weighting function:
* Prelec’s (1992) probability weighting function:
* Regret (or rejoice) function:

Table A3. Summary of intertemporal decision models.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ID | Model | Category | Authors | Year | Source (Journal or Book) |
| 1 | Exponential | Delay discounting | Samuelson | 1937 | Review of Economic Studies |
| 2 | Hyperbolic | Delay discounting | Mazur | 1987 | The Effect of Delay and Intervening Events on Reinforcement Value |
| 3 | Hyperbolic w/ power time (Mazur) | Delay discounting | Mazur | 1987 | The Effect of Delay and Intervening Events on Reinforcement Value |
| 4 | Generalized hyperbolic | Delay discounting | Loewenstein and Prelec | 1992 | Quarterly Journal of Economics |
| 5 | Exponential time | Delay discounting | Roelofsma | 1996 | Acta Psychologica |
| 6 | Quasi-hyperbolic | Delay discounting | Laibson | 1997 | Quarterly Journal of Economics |
| 7 | Hyperbolic w/ power denominator | Delay discounting | Green and Myerson | 2004 | Psychological Bulletin |
| 8 | Hyperbolic w/ power time (Rachlin) | Delay discounting | Rachlin | 2006 | Journal of the Experimental Analysis of Behavior |
| 9 | Constant sensitivity | Delay discounting | Ebert and Prelec | 2007 | Management Science |
| 10 | Double exponential | Delay discounting | McClure et al. | 2007 | Journal of Neuroscience |
| 11 | Fixed cost | Delay discounting | Benhabib et al. | 2010 | Games and Economic Behavior |
| 12 | Generalized hyperbolic w/ increasing elasticity | Delay discounting | Scholten et al. | 2014 | Cognitive Science |
| 13 | Dual systems | Delay discounting | Loewenstein et al. | 2015 | Decision |
| 14 | Interval | Interval discounting | Read | 2001 | Journal of Risk and Uncertainty |
| 15 | Common aspect attenuation | Interval discounting | Green et al. | 2005 | Journal of Experimental Psychology: Learning, Memory & Cognition |
| 16 | Generalized interval | Interval discounting | Scholten and Read | 2006 | Management Science |
| 17 | As-soon-as-possible | Interval discounting | Kable and Glimcher | 2010 | Journal of Neurophysiology |
| 18 | Generalized interval w/ increasing elasticity | Interval discounting | Scholten et al. | 2014 | Cognitive Science |
| 19 | Similarity w/ difference | Time-as-attribute | Leland | 2002 | Economic Inquiry |
| 20 | Similarity w/ ratio | Time-as-attribute | Leland | 2002 | Economic Inquiry |
| 21 | Additive utility | Time-as-attribute | Killeen | 2009 | Psychological Review |
| 22 | Tradeoff model | Time-as-attribute | Scholten and Read | 2010 | Psychological Review |
| 23 | DRIFT | Time-as-attribute | Read et al. | 2013 | Journal of Experimental Psychology: Learning, Memory & Cognition |
| 24 | Attribute-based model w/ power transformations | Time-as-attribute | Dai and Busemeyer | 2014 | Journal of Experimental Psychology: General |
| 25 | Generalized tradeoff model | Time-as-attribute | Scholten et al. | 2014 | Cognitive Science |
| 26 | Intertemporal choice heuristics | Time-as-attribute | Ericson et al. | 2015 | Psychological Science |
| 27 | Proportional difference | Time-as-attribute | Cheng and González-Vallejo | 2016 | Decision |

Table A4. Functional forms of intertemporal decision models. The notations are designed for choices between and , where ,. . denotes the utility or choice propensity of *X* and denotes the utility or choice propensity of *Y*. For delay discounting models, is not presented but can be obtained by replacing and in with and . For time-as-attribute models that represent options’ advantages on an ordinal scale, denotes the argument for option *X* and denotes the argument for option *Y*. Free parameters are denoted by Greek letters, with corresponding domains and prior distributions shown in Supplementary Table 5.

|  |  |  |  |
| --- | --- | --- | --- |
| ID | Model | Function | Stochastic specification |
| 1 | Exponential |  | Logit or Probit |
| 2 | Hyperbolic |  | Logit or Probit |
| 3 | Hyperbolic w/ power time (Mazur) |  | Logit or Probit |
| 4 | Generalized hyperbolic |  | Logit or Probit |
| 5 | Exponential time[[5]](#footnote-5) |  | Logit or Probit |
| 6 | Quasi-hyperbolic |  | Logit or Probit |
| 7 | Hyperbolic w/ power denominator |  | Logit or Probit |
| 8 | Hyperbolic w/ power time (Rachlin) |  | Logit or Probit |
| 9 | Constant sensitivity |  | Logit or Probit |
| 10 | Double exponential |  | Logit or Probit |
| 11 | Fixed cost |  | Logit or Probit |
| 12 | Generalized hyperbolic w/ increasing elasticity |  | Logit or Probit |
| 13 | Dual systems |  | Logit or Probit |
| 14 | Interval |  | Logit or Probit |
| 15 | Common aspect attenuation |  | Logit or Probit |
| 16 | Generalized interval |  | Logit or Probit |
| 17 | As-soon-as-possible |  | Logit or Probit |
| 18 | Generalized interval w/ increasing elasticity |  | Logit or Probit |
| 19 | Similarity w/ difference |  | Constant error |
| 20 | Similarity w/ ratio |  | Constant error |
| 21 | Additive utility |  | Logit or Probit |
| 22 | Tradeoff model |  | Logit or Probit |
| 23 | DRIFT |  | Logit or Probit |
| 24 | Attribute-based model w/ power transformations |  | Logit or Probit |
| 25 | Generalized tradeoff model |  | Logit or Probit |
| 26 | Intertemporal choice heuristics |  | Logit or Probit |
| 27 | Proportional difference |  | Logit or Probit |

*Note*. In order to ensure that the KL divergence between two series of model predictions is tractable, choice probabilities for all models are bounded within the interval [0.001, 0.999]. The following additional functions are used in Supplementary Table 4:

* is an indicator function that returns 1 if the argument is true, and 0 otherwise.
* Power value function: .
* Increasingly elastic value function as in Scholten et al. (2014): .

Table A5. Parameter bounds and prior distributions for both risky and intertemporal decision models.

|  |  |  |
| --- | --- | --- |
| Parameter | Domain | Prior distribution |
|  | [0, +∞) | Exponential (rate = 1) |
|  | [0, 1] | U(0, 1) |
|  | [0.5, 1] | U(0.5, 1) |
|  | [-2, 2] | U(-2, 2) |
|  | (-∞, +∞) | N(0, 1) |
|  | Integer, [0, 49] | Binomial (49, 0.5) |

*Note*. has the domain of [0, +∞), meaning that the domain of is [1, +∞). Similarly, hasthe domain of [0, 49] (integer), meaning that the domain of is [0, 49] (integer).

1. NA is the abbreviation of “not applicable”, meaning that the model itself involves a stochastic specification. [↑](#footnote-ref-1)
2. This is a “special” TAX model assuming that all weight transfers are the same fixed proportion of the branch giving up weight (Birnbaum, 2008; pp. 470, Eq.8a). [↑](#footnote-ref-2)
3. This is a simplification of the original (simulation based) utility-weighted sampling theory. [↑](#footnote-ref-3)
4. (with ) represents the target value in this model and other target-related models. [↑](#footnote-ref-4)
5. The original exponential time discounting model uses log(*t*) to transform delay *t*. Since this function cannot adequately accommodate *t* = 0, we have replaced it with in line with the specification of Scholten et al. (2014). This revision has made this model mathematically equivalent to Loewenstein and Prelec’s (1992) generalized hyperbolic discounting model because . [↑](#footnote-ref-5)